codon substitution models and the analysis of natural selection pressure

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Details matter!

Codon models: waaaaay **too much** to cover in this talk

Macro-evolutionary inference of selection intensity:

- Very complex and diverse modelling strategies
- Deep statistical issues
- Model testing and interpretation
- Strong opinions about "the right thing to do"



Chapter 13

Looking for Darwin in Genomic Sequences: Validity and Success Depends on the Relationship Between Model and Data

Christopher T. Jones, Edward Susko, and Joseph P. Bielawski

Abstract

Check for

Codon substitution models (CSMs) are commonly used to infer the history of natural section for a set of protein-coding sequences, often with the explicit goal of detecting the signature of positive Darwinian selection. However, the validity and success of CSMs used in conjunction with the maximum likelihood (ML) framework is sometimes challenged with claims that the approach might too often support false conclusions. In this chapter, we use a case study approach to identify four legitimate statistical difficulties associated with inference of evolutionary events using CSMs. These include: (1) model misspecification, (2) low information content, (3) the confounding of processes, and (4) phenomenological load, or PL. While past criticisms of CSMs can be connected to these issues, the historical critiques were often misdirected, or overstated, because they failed to recognize that the success of any model-based approach depends on the relationship between model and data. Here, we explore this relationship and provide a candid assessment of the limitations of CSMs to extract historical information from extant sequences. To aid in this assessment, we provide a brief overview of: (1) a more realistic way of thinking about the process of codon evolution framed in terms of population genetic parameters, and (2) a novel presentation of the ML statistical framework. We then divide the development of CSMs into two broad phases of scientific activity and show that the latter phase is characterized by increases in model complexity that can sometimes negatively impact inference of evolutionary mechanisms. Such problems are not yet widely appreciated by the users of CSMs. These problems can be avoided by using a model that is appropriate for the data; but, understanding the relationship between the data and a fitted model is a difficult task. We argue that the only way to properly understand that relationship is to perform in silico experiments using a generating process that can mimic the data as closely as possible. The mutation-selection modeling framework (MutSel) is presented as the basis of such a generating process. We contend that if complex CSMs continue to be developed for testing explicit mechanistic hypotheses, then additional analyses such as those described in here (e.g., penalized LRTs and estimation of PL) will need to be applied alongside the more traditional inferential methods.

Key words Codon substitution model, dN/dS, False positives, Maximum likelihood, Mechanistic model, Model misspecification, Mutation-selection model, Parameter confounding, Phenomenological load, Phenomenological model, Positive selection, Reliability, Statistical inference, Site-specific fitness landscape

Maria Anisimova (ed.), Evolutionary Genomics: Statistical and Computational Methods, Methods in Molecular Biology, vol. 1910, https://doi.org/10.1007/978-1-4939-9074-0_13, © The Author(s) 2019

Details: **book chapter** (PDF) on course website

1. mechanistic codon models

I. mechanistic codon models



reconciling evolutionary time scales







- Wright-Fisher population
- drift: **N**
- mutation: μ
- selection: **s**_{ij}
- s_{ij} vary among sites AND amino acids



- **realism**: fixation probability depends on fitness of ancestral and derived amino acids in the context of the protein.
- the cost of realism: usually too complex to fit such a model to real data (caveat: some versions will allow new ways to analyze big datasets)



population genetics of natural selection at a single codon site (h)

 $f^h = \left\langle f_1 , \dots, f_{61} \right\rangle$ $s_{ij}^h = f_j^h - f_i^h$ selection coefficients

fitness coefficients

 $\Pr(s_{ij}^{h}) = \frac{2s_{ij}^{h}}{1 - e^{-4Ns_{ij}^{h}}}$ fixation probability (Kimura, 1962)

2. phenomenological codon models









- phenomenological parameters
- ts/tv ratio: *к*
- codon frequencies: π_j
- $\boldsymbol{\omega} = dN/dS$
- parameter estimation via ML
- stationary process



Goldman and Yang (1994) Muse and Gaut (1994)

the instantaneous **rate matrix**, *Q*, is very big: 61 × 61



Phenomenological codon models: just a few parameters can cover the 3721 changes between codons!

Context $(i \rightarrow j)$ should matter... But, the ω is always the same!

				to codo	on below:			
From codon below:	TTT (Phe)	TTC (Phe)	TTA (Leu)	ΠG (Leu)	CTT (Leu)	CTC (Leu)	·····Þ	GGG (Gly)
TTT (Phe)		$\kappa\pi_{\mathrm{TTC}}$	$\omega \pi_{\mathrm{TTA}}$	$\omega \pi_{\mathrm{TTG}}$	$\omega\kappa\pi_{\mathrm{TTT}}$	0		0
TTC (Phe)	$\kappa \pi_{\mathrm{TTT}}$		$\omega \pi_{\mathrm{TTA}}$	$\omega \pi_{\mathrm{TTG}}$	0	$\omega \kappa \pi_{\mathrm{CTC}}$		0
TTA (Leu)	$\omega \pi_{\mathrm{TTT}}$	$\omega \pi_{\mathrm{TTC}}$			0	0		0
ΠG (Leu)	$\omega \pi_{\mathrm{TTT}}$	$\omega \pi_{\mathrm{TTC}}$	$\kappa \pi_{\mathrm{TTA}}$		0	0		0
CTT (Leu)	$\omega\kappa\pi_{\mathrm{TTT}}$	0	0	0		$\kappa \pi_{\mathrm{CTC}}$		0
CTC (Leu)	0	$\omega \kappa \pi_{\mathrm{TTC}}$	0	0	$\kappa\pi_{ m TTT}$			0
¥	▼	•		•	▼	•	******	
GGG (Gly)	0	0	0	0	0	0	0	

* This is equivalent to the codon model of Goldman and Yang (1994). Parameter $\boldsymbol{\omega}$ is the ratio d_N/d_S , $\boldsymbol{\kappa}$ is the transition/transversion rate ratio, and $\boldsymbol{\pi}_i$ is the equilibrium frequency of the target codon (*i*).

probability of substitution between codons over time, P(t)



likelihood of these data at a site given a model



note: analysis is typically done by using an unrooted tree

likelihood of the data at all sites given a model



3. bridging selection between time-scales





Two explicit ways to reconcile **population genetics** and **macroevolution**:

1. map fitness to equilibrium frequencies

2. expected index of selection intensity



(1) Sella and Hirsh 2005; (2) Jones et al. 2016

1. fitness coefficients map to stationary codon frequencies



(Sella and Hirsh 2005)



- $dN/dS = \omega$ when matrix A^h is replaced by matrix Q of model M0
- *dN/dS* is an analog of *w* under MutSel

4. three positive selection scenarios





1932: adaptive landscapes and "shifting balance"



Sewall Wright

o introduces "ADAPTIVE LANDSCAPE" as a metaphor



 introduces "SHIFTING BALANCE" as a model (SBT more complex than I will present) positive selection: 3 evolutionary scenarios



frequency dependent selection



episodic adaptation

dynamic fitness landscape



non-adaptive shifting balance static fitness landscape



¹ frequency-dependent adaptive landscape (weird)



1 frequency-dependent adaptive landscape (weird)

- 1. amino acid at a site has f^h ; all others have $f^h + s$
- 2. fitness values swap when a substitution occurs



MutSelM0: (1) and (2) above imply Markov chain properties with the same rate matrix Q as **codon model M0**

¹ frequency-dependent adaptive landscape (weird)



conclusion: phenomemologcial codon models assume frequency-dependent selection

[dos Reis (2015); Jones et al. (2016)]





optimal function in a stable environment



population: at fitness peakfitness peak: stationaryFFTNS: keeps population at peak



sub-optimal function in a novel environment



population: lower fitness
fitness peak: moving
FFTNS: increase population mean fitness
(non-stationary process)



episodic adaptive evolution of a novel function



population: returns to peak
fitness peak: stabilized
FFTNS: increases population mean
fitness until at peak

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Research



Cite this article: dos Reis M. 2015 How to calculate the non-synonymous to synonymous rate ratio of protein-coding genes under the Fisher – Wright mutation – selection framework. *Biol. Lett.* **11**: 20141031. http://dx.doi.org/10.1098/rsbl.2014.1031

Received: 8 December 2014 Accepted: 16 March 2015

Molecular evolution

How to calculate the non-synonymous to synonymous rate ratio of protein-coding genes under the Fisher – Wright mutation – selection framework

Mario dos Reis

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First principles of population genetics are used to obtain formulae relating the non-synonymous to synonymous substitution rate ratio to the selection coefficients acting at codon sites in protein-coding genes. Two theoretical cases are discussed and two examples from real data (a chloroplast ge

polymerase) are given. The formulae give much insight into the non-synonymous substitutions and may inform the development to detect adaptive evolution.

4. The non-synonymous rate during adaptive evolution





conclusion : episodic models "work" because w>1 is a consequence of a system moving towards a new fitness peak.

conclusion : episodic models "work" because they are sensitive to non-stationary behavior

[dos Reis (2015); Jones et al. (2016)]





3 shifting balance: movement around stationary peak (non-adaptive)



mutation and drift can move a pop. off a fitness peak

shifting balance: the MutSel landscape (Jones et al. 2016)



3

3 shifting balance: positive selection on a MutSel landscape



conclusion: $p_+ > 0$ as long as number of viable amino acids > 1 at a site



p+ = positive selection without adaptation (maintenance!)

p- = related to "fixed drift load"



 dN^{h}/dS^{h} depends on the current amino acid



conclusion: positive selection operates on a stationary fitness peak in the same way as when there is an adaptive peak shift



5. nearly-neutral theory and "heterotachy"

dN^{h}/dS^{h} depends on the current amino acid



shifting balance: a mechanistic model



What does heterotachy "look" like on a tree?



Different levels of shifting balance over a landscape





6. some common types of codon models

 $\mathcal{Q}_{ij} = \begin{cases} 0 & \text{if } i \text{ and } j \text{ differ by } > 1 \\ \pi_j & \text{for synonymous tv.} \\ \kappa \pi_j & \text{for synonymous ts.} \\ \omega \pi_j & \text{for non-synonymous tv.} \\ \omega \kappa \pi_j & \text{for non-synonymous ts.} \end{cases}$ Goldman and Yang (1994) Muse and Gaut (1994)

this codon model "MO"





same *w* for all branches



two basic types of models...



branch models (*w* varies among branches)



site models (*w* varies among sites)



branch models*



variation (ω) among branches:	approach
Yang, 1998	fixed effects
Bielawski and Yang, 2003	fixed effects
Seo et al. 2004	auto-correlated rates
Kosakovsky Pond and Frost, 2005	genetic algorithm
Dutheil et al. 2012	clustering algorithm

* these methods can be useful when selection pressure is strongly **episodic and** functional change is substantial

site models*

GTG	CTG	TCT	CCT	GCC	GAC	AAG	ACC	AAC	GTC	AAG	GCC	GCC	TGG	GGC	AAG	\mathbf{GTT}	GGC	GCG	CAC
			G.C				т	т										. GC	A
• • •	• • •		C	т	• • •	• • •	• • •	• • •	Α		A.T			. AA	• • •	A.C	• • •	AGC	
	c		G.A	. AT		A			Α		AA.	TG.		G		Α	т	.GC	т
• • •	c	G	GA.	т		• • •	т	с	G	A	• • •	AT.		т	• • •	G	A	.GC	

variation (ω) among sites:	approach	This is NOT a comprehensive list!
Yang and Swanson, 2002	fixed effects (ML)	
Bao, Gu and Bielawski, 2006	fixed effects (ML)	
Massingham and Goldman, 2005	site wise (LRT)	
Kosakovsky Pond and Frost, 2005	site wise (LRT)	
Nielsen and Yang, 1998	mixture model (ML)	
Kosakovsky Pond, Frost and Muse, 2005	mixture model (ML)	
Huelsenbeck and Dyer, 2004; Huelsenbeck et al. 2006	mixture (Bayesian)	
Rubenstein et al. 2011	mixture model (ML)	
Bao, Gu, Dunn and Bielawski 2008 & 2011	mixture (LiBaC/MBC)	
Murell et al. 2013	mixture (Bayesian)	

• useful when at some sites evolve under **diversifying selection** pressure over long periods of time

site models: discrete mixture model (M3)



 $\omega_0 = 0.01 \quad \omega_1 = 1.0 \quad \omega_2 = 2.0$

interpretation of a sites-model

Powerful approach for antagonistic co-evolution



models for variation among branches & sites



branch-site models (combines the features of above models)

models for variation among branches & sites

		This	s is NOT a ehensive list!
variation (ω) among branches & sites:	approach	compi	
Yang and Nielsen, 2002	fixed+mixture (ML)		
Forsberg and Christiansen, 2003	fixed+mixture (ML)		
Bielawski and Yang, 2004	fixed+mixture (ML)		
Giundon et al., 2004	covarion-like (ML)		
Zhang et al. 2005	fixed+mixture (ML)		
Kosakovsky Pond et al. 2011, 2012	full mixture (ML)		
Jones et al., 2016, 2018, 2020	mix-covarion-like (ML)	

* these methods can be useful when selection **pressures change over** time at just a fraction of sites

* it can be a challenge to apply these methods properly



two scenarios can yield branch-sites with $d_N/d_S > 1$



7. "bells –n– whistles"...

codon models + "other brocesses."

		to codon below:										
From codon below:	TTT (Phe)	TTC (Phe)	TTA (Leu)	ΠG (Leu)	CTT (Leu)	CTC (Leu)		GGG (Gly)				
TTT (Phe)		$\kappa \pi_{\rm TTC}$	$\omega \pi_{\text{TTA}}$	$\omega \pi_{TTG}$	$\omega \kappa \pi_{TTT}$	0		0				
TTC (Phe)	κπτττ		$\omega \pi_{\text{TTA}}$	$\omega \pi_{\rm TTG}$	0	ωκπετε		0				
TTA (Leu)	$\omega \pi_{TTT}$	$\omega \pi_{\text{TTC}}$			0	0		0				
TTG (Leu)	$\omega \pi_{\rm TTT}$	$\omega \pi_{\rm TTC}$	$\kappa \pi_{\rm TTA}$		0	0		0				
CTT (Leu)	$\omega\kappa\pi_{\rm TTT}$	0	0	0		$\kappa \pi_{\rm CTC}$		0				
CTC (Leu)	0	$\omega \kappa \pi_{TTC}$	0	0	$\kappa \pi_{\rm TTT}$			0				
GGG (Gly)	0	0	0	0	0	0	0					

TIC (Phe) xxTTT ouxTTA ouxTTA ouxTTA ouxTTA TIA (Leu) ouxTTT ouxTTA 0 0 0 TIC (Leu) ouxTTT ouxTTA 0 0 CTT (Leu) ouxTTT ouxTTA 0 0	0	0	0	0	0	0	0	GGG (Gly)
ПС (Phe) хгтт шигда шигда шигда 0 шигда ПА (teu) шигда шгда 0 0 0 0 Пб (teu) шигда шгда — 0 0 0 1 СП (teu) шигда 0 0 0 — игддсс	0		KTTTT	0	0	WKTTTC	0	CTC (Leu)
TC (Phe) xTTT unTta unta unta <thunta< th=""> <thunta< th=""> unta</thunta<></thunta<>	0	KXCTC		0	0	0	OKWTIT	СП (Leu)
ПС (Phe) клада — оклада оклада 0 оклада. ПА (Leu) οκεμαι σκατια — 0 0					KWTTA	00 TTTC	OWTIT	TTG (Leu)
								TTA (Leu)

"bells –n– whistles"... some general categories

- 1. alternative models of codon frequencies
- 2. GTR process at DNA-level
- 3. among-site synonymous rate (d_s) variation
- 4. double & triple nucleotide changes
- 5. amino acid exchangeabilities
- 6. multi-process variation among sites
- 7. multi-pattern (tree) variation among sites

(the π 's parameters are important)

(NOT a mutational process; small effects)

(important for some genes)

(confounded with heterotachy)

(confounded with codon frequencies via fitness)

(do we really want this much complexity?)

(this can be important; e.g., recombination)

				to code	on below:			
From codon below:	TTT (Phe)	TTC (Phe)	ΠΑ (Leu)	TIG (Leu)	CTT (Leu)	CTC (Leu)		GGG (Gly)
TTT (Phe)		$\kappa \pi_{\rm TTC}$	$\omega \pi_{\text{TTA}}$	$\omega \pi_{\text{TTG}}$	$\omega\kappa\pi_{\rm TTT}$	0		0
TTC (Phe)	$\kappa \pi_{\rm TTT}$		$\omega \pi_{TTA}$	$\omega \pi_{TTG}$	0	$\omega \kappa \pi_{\rm CTC}$		0
TTA (Leu)	$\omega \pi_{\rm TTT}$	$\omega \pi_{\text{TTC}}$			0	0		0
TTG (Leu)	$\omega \pi_{\rm TTT}$	$\omega \pi_{\text{TTC}}$	$\kappa \pi_{\rm TTA}$		0	0		0
CTT (Leu)	$\omega \kappa \pi_{\rm TTT}$	0	0	0		$\kappa \pi_{\rm CTC}$		0
CTC (Leu)	0	$\omega \kappa \pi_{\text{TTC}}$	0	0	$\kappa \pi_{\rm TTT}$			0
GGG (Gly)	0	0	0	0	0	0	0	

Is adding more "bells –n– whistles" the way forward?

8. A new approach...

Phenotype-Genotype codon models



Phenotype-Genotype codon models... PhyloG2P

Trends in Ecology & Evolution



Opinion

Phylogenetics is the New Genetics (for Most of Biodiversity)

Stacey D. Smith, 1,6,*,@ Matthew W. Pennell,² Casey W. Dunn,³ and Scott V. Edwards^{4,5}

Despite substantial progress in understanding the genetic basis for differences in morphology, physiology, and behavior, many phenotypes of interest are difficult to study with traditional genetic approaches because their origin traces to deep nodes in the tree of life. Moreover, many species are not amenable to either large-scale sampling or laboratory crosses. We argue that phylogenetic methods and theory provide tremendous power to identify the functional genetic variation underlying trait evolution. We anticipate that existing statistical comparative approaches will be more commonly applied to studying the genetic basis for phenotypic evolution as whole genomes continue to populate the tree of life. Nevertheless, new methods and approaches will be needed to fully capitalize on the power of clade-scale genomic datasets.

One of the fundamental goals of biology is to connect variation across genomes to differences in

phenotypes. With advances in sequencing and molecular genetic techniques, this area of biology

has blossomed in recent years, revealing the genetic basis for traits ranging from floral scent [1] to

sociality [2] to herbivory [3]. At the same time, statistical methods for analyzing these data have

also proliferated [4-6]. At their core, however, all classical and population genetic methods for

genotype-to-phenotype mapping (see Glossary) work by associating genetic variation with

differences in the trait of interest. Thus, they require a population with segregating phenotypic variation, which could be produced artificially through crosses or mutagenesis or could occur naturally, such as in polymorphic species or hybrid zones between species. As with any statistical approach, association methods [e.g., genome-wide association studies (GWASs)] have sig-

nificant challenges and pitfalls [6,7]. Still, the loci uncovered by association mapping and similar

methods have often been validated in subsequent functional studies [8,9], confirming their ability

Despite the success of this population genetic program for genotype-phenotype mapping, it

presents significant limitations for understanding the genetic basis of phenotypes for most of

biodiversity. First, many species cannot be propagated artificially or sampled in the wild at the

of interest). Second, and more importantly, many traits of interest are not found segregating in

nature nor can different species with contrasting phenotypes be crossed. For example, mammals

segregating for pouches. As a consequence, our understanding of the genetic basis for phenotypic diversity is concentrated around a narrow range of species and traits - often those that vary in model organisms amenable to genetic studies. Although loci discovered through

genetic studies of model species often later help to explain variation at deeper phylogenetic levels

inverted (Figure 1). We suggest, and recent studies confirm, that beginning from a phylogenetic

Most of Biodiversity Is Beyond the Reach of Classical Genetics

to identify regions of the genome that contribute to phenotypic differences.

Highlights

Genome sequencing is rapidly spreading beyond model organisms, opening the door to comparative studies that can reveal the genetic basis for phenotypic variation across species. Nevertheless, statistical comparative methods have not been frequently applied to these data.

New phylogenetic methods have been developed with the explicit goal of linking genes and even specific mutations to species differences ('PhyloG2P'), Applications of these methods show great promise for uncovering new sources of functional variation and tackling traits beyond the reach of traditional genetic approaches.

Parallel advances in statistical comparative methods present new avenues for expanding the phylogenetic toolkit and creating tailored approaches for mapping genotype to phenotype.

¹Department of Ecology and Evolutionary Biology, University of Colorado, Boulder, CO 80309, USA ²Department of Zoology and Biodiversity Research Centre, University of British Columbia, Vancouver, BC V6T 1Z4, Canada ³Department of Ecology and Evolutionary Biology, Yale University, New Haven, CT 06520, USA Department of Organismic and scale needed for association mapping (usually hundreds of individuals, depending on the trait Evolutionary Biology, Harvard University, Cambridge, MA 02138, USA ⁵Museum of Comparative Zoology Harvard University, Cambridge, with and without pouches cannot be crossed, precluding the creation of a mapping population MA 02138, USA ⁶http://www.colorado.edu/smithlab

*Correspondence: Stacey.D.Smith@colorado.edu (i.e., across species [10,11]), we wonder what we might discover if this research program were (S.D. Smith). [@]Twitter: @iochromaland (S.D. Smith).

Check for updates

Trends in Ecology & Evolution, May 2020, Vol. 35, No. 5 https://doi.org/10.1016/j.tree.2020.01.005 415 © 2020 Elsevier I td. All rights reserved.

Phenotype only models:

Cornwell, W. and Nakagawa, S. 2017. Phylogenetic comparative methods. Curr.Biol., 27: 327–338. phenotype models

Phenotype + Genotype models:

Mayrose, I. and Otto, S. P. (2011). A likelihood method for detecting trait-dependent shifts in the rate of molecular evolution. Mol. Biol. Evol., 28: 759-770.	+ DNA model
Lartillot, N. and Poujol, R. (2011). A phylogenetic model for investigating correlated evolution of substitution rates and continuous phenotypic characters. Mol. Biol. Evol., 28: 729-744.	+ codon model
O'Connor, T. D. and Mundy, N. I. (2013). Evolutionary modeling of geneotype-phenotype association and application to the primate coding and non-coding mtdna rate variation. Evolutionary Bioinformatics, 9: 301-316.	+ DNA model
Karin, E. L., Wicke, S., Pupko, T., and Mayrose, I. (2017). <i>An integrated model of phenotypic trait changes and site-specific sequence evolution</i> . Syst. Biol., 66: 917-933.	+ DNA model
Jones, C. T., Youssef, N., Susko, E., & Bielawski, J. P. (2020). A Phenotype-Genotype Codon Model for Detecting Adaptive Evolution. Systematic Biology, 69(4), 722-738.	+ codon model
Halabi, K., Karin, E. L., Guéguen, L., & Mayrose, I. (2021). A codon model for associating phenotypic traits with altered selective patterns of sequence evolution. Systematic Biology, 70(3), 608-622.	+ codon model

Background for Jones et al. (2020) PG model...

We can model heterotachy with a covarion model

We detect "SB signal" with covarion codon models!



[Guindon et al., (2004); Jones et al. (2016); Jones et al. (2018); Jones et al. 2019]







(any similarities = coincidence)

P-G MODEL: dN/dS linked to phenotype change ω ω ω ω ω (A)gene evolution phenotype evolution

fraction of sites = gene evolution depends on changes in phenotype

Jones, C. T., Youssef, N., Susko, E., & Bielawski, J. P. (**2020**). A Phenotype-Genotype Codon Model for Detecting Adaptive Evolution. Systematic biology, 69(4), 722-738.



phenotype mapping 1 (of many)

gene evolution

e	notype + Genotype models:	
	Mayrose, I. and Otto, S. P. (2011). A likelihood method for detecting trait-dependent shifts in the rate of molecular evolution. Mol. Biol. Evol., 28: 759-770.	+ DNA model
	Lartillot, N. and substitution ran Detect adaptive molecular evolution	+ codon model
	O'Connor, T. E association an Evolutionary B (possibly without $d_N/d_S > 1$)	+ DNA model
	Karin, E. L., Wicke, S., Pupko, T., and Mayrose, I. (2017). <i>An integrated model of phenotypic trait changes and site-specific sequence evolution</i> . Syst. Biol., 66: 917-933.	+ DNA model
	Jones, C. T., Youssef, N., Susko, E., & Bielawski, J. P. (2020). A Phenotype-Genotype Codon Model for Detecting Adaptive Evolution. Systematic biology, 69(4), 722-738.	+ codon model
	Halabi, K., Karin, E. L., Guéguen, L., & Mayrose, I. (2021). A codon model for associating phenotypic traits with altered selective patterns of sequence evolution. Systematic Biology, 70(3), 608-622.	+ codon model

Phenotype only models:

Cornwell, W. and Nakagawa, S. 2017. Phylogenetic comparative methods. Curr.Biol., 27: 327-338.

Phe

phenotype models

9. model-based inference



You will get "the basics" in the evening **PAML-Lab**